

# Causal Discovery and Network Inference for Biological and Biomedical Data

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CNRS, Institut Curie & Prairie Institute, PSL University, Paris, France

Eclaire GT, Kickoff Meeting, LIP6, Sorbonne University

24 March 2026





# Correlation *versus* Information

$$\sigma_{x,y} = \frac{1}{N} \sum_i^N (x_i - \bar{x})(y_i - \bar{y})$$

$X, Y$  need to be continuous

$$I(X; Y) = \sum_{x,y} p(x, y) \log_2 \frac{p(x, y)}{p(x)p(y)}$$

$X, Y$  can be categorical, continuous or even high-dimensional  
(sequences, images,...)

Identical binary variables:  $I(X; Y) = 1$  bit

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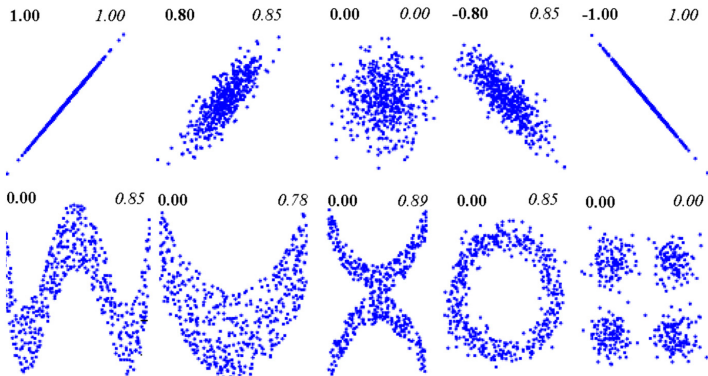
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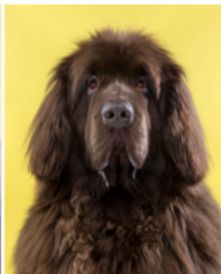
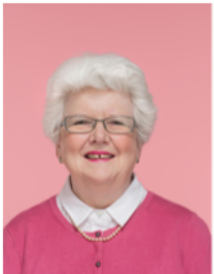
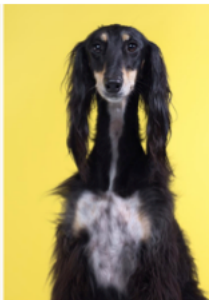
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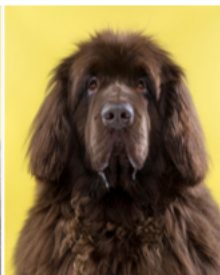
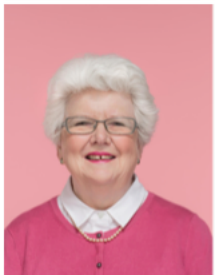
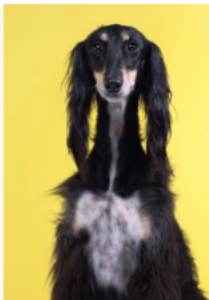
**Correlation**    *Information*



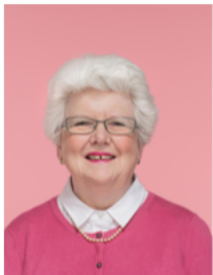
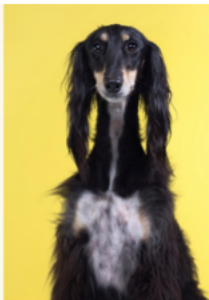
How much is  $I(\text{master}; \text{dog})$  in bits ?



Or... how much is  $I(\text{cell image}; \text{cell omics})$  in bits ?



Or... how much is  $I(\text{cell image}; \text{cell omics})$  in bits ?



but... why should we care about information ??

# Likelihood, Cross-Entropy and Cross-Information

## Likelihood $q(\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(N)})$ and Cross-Entropy $H(p, q)$

Given some Data,  $N$  i.i.d. samples  $\sim p(\mathbf{x})$ , and a Model distribution,  $q(\mathbf{x})$ ,  
The probability that the sequence  $\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \dots, \mathbf{x}^{(N)}$  was generated by  $q(\mathbf{x})$  is:

$$q(\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(N)}) \rightarrow e^{-N H(p, q)} = e^N \sum_{\mathbf{x}} p(\mathbf{x}) \log q(\mathbf{x}) = e^{-N(H(p) + D_{\text{KL}}(p||q))}$$

where  $D_{\text{KL}}(p||q) = \sum_{\mathbf{x}} p(\mathbf{x}) \log \frac{p(\mathbf{x})}{q(\mathbf{x})} \geq 0$  is the Kullback-Leibler divergence

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**Proof:**  $\frac{1}{N} \log q(\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \dots, \mathbf{x}^{(N)}) = \frac{1}{N} \sum_i^N \log q(\mathbf{x}^{(i)}) \rightarrow \sum_{\mathbf{x}} p(\mathbf{x}) \log q(\mathbf{x})$  □

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**Multivariate (cross-)entropy**  $H(X_1, \dots, X_n) \equiv H(p, q) = - \sum_{\{\mathbf{x}_i\}_n} p(\{\mathbf{x}_i\}_n) \log q(\{\mathbf{x}_i\}_n)$

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**Multivariate (cross-)information**  $I(X_1; \dots; X_n)$  are defined from  $H(X_1, \dots, X_n)$  and vice versa

$$I(X; Y) = H(X) + H(Y) - H(X, Y)$$

$$I(X; Y; A) = H(X) + H(Y) + H(A) - H(X, Y) - H(X, A) - H(Y, A) + H(X, Y, A)$$

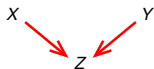
$$I(X_1; \dots; X_n) = - \sum_{S \subseteq V} (-1)^{|S|} H(S)$$

$$H(X_1, \dots, X_n) = - \sum_{S' \subseteq V} (-1)^{|S'|} I(S') \quad [\text{Inclusion - Exclusion Principle}]$$

# Link between constraint-based and score-based methods

Constraints  $\equiv$  Cancelling (combinations of) multivariate cross-informations

## V-structure

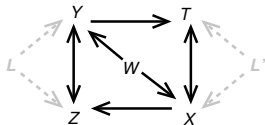


$$\begin{aligned} I(X; Y) &= 0 \\ I(X; Z) &> 0 \\ I(Z; Y) &> 0 \\ I(X; Z; Y) &< 0^* \end{aligned}$$

## Latent variables L



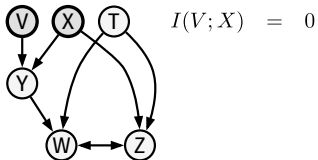
$$\begin{aligned} I(X; Y) &= I(X; W) = I(W; Z) = 0 \\ I(X; Z) &> 0 \\ I(Z; Y) &> 0 \quad I(Y; W) &> 0 \\ I(X; Z; Y) &< 0^* \quad I(Z; Y; W) &< 0^* \\ I(X; Z; W) &= 0 \quad I(X; Y; W) &= 0 \\ I(X; Z; Y; W) &> 0 \end{aligned}$$



\* **Signature of Causality** from MI decomposition:  $I(X; Y) = I(X; Z; Y) + I(X; Y|Z)$

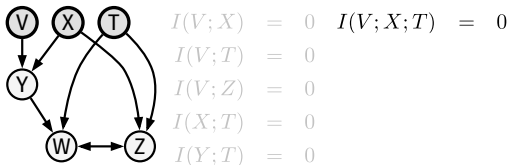
# Constraints in Ancestral Graphs

## Information mutuelle nulle



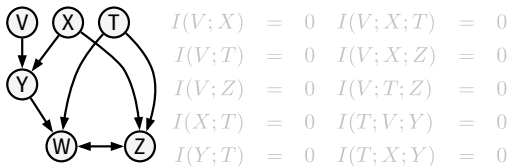
# Constraints in Ancestral Graphs

## Interaction nulle

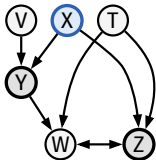


# Constraints in Ancestral Graphs

## Information et interaction nulle



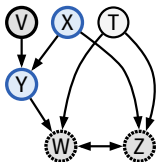
## Information mutuelle conditionnelle nulle



$$I(Y; Z | X) = I(Y; Z) - I(Y; Z; X) = 0$$

# Constraints in Ancestral Graphs

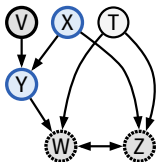
## Interaction conditionnelle nulle



$$I(V; \{W, Z\} | X, Y) = I(V; W | X, Y) + I(V; Z | W, X, Y)$$

# Constraints in Ancestral Graphs

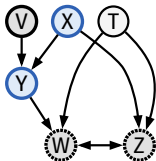
## Interaction conditionnelle nulle



$$\begin{aligned} I(V; \{W, Z\} \mid X, Y) &= I(V; W \mid X, Y) + I(V; Z \mid W, X, Y) \\ &= I(V; W) - I(V; W; X) - I(V; W; Y) + I(V; W; X; Y) \\ &\quad + I(V; Z) - I(V; Z; X) - I(V; Z; Y) - I(V; Z; W) \\ &\quad + I(V; Z; X; Y) + I(V; Z; X; W) + I(V; Z; Y; W) - I(V; Z; X; Y; W) \end{aligned}$$

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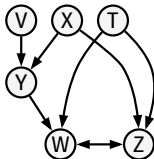
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# Constraints in Ancestral Graphs

## Contraintes : termes informatifs nuls



$$I(V; X) = 0, I(V; T) = 0, I(V; Z) = 0$$

$$I(X; T) = 0, I(Y; T) = 0$$

$$I(V; X; T) = 0, I(V; X; Z) = 0, I(V; T; Z) = 0$$

$$I(T; V; Y) = 0, I(T; X; Y) = 0$$

$$I(Y; Z | X) = 0, I(X; W | Y) = 0, I(V; W | Y) = 0$$

$$I(V; \{W, Z\} | X, Y) = 0$$

# Likelihood of Ancestral Graphs (incl $\rightarrow$ and $\leftrightarrow$ )

## Theorem: likelihood of ancestral graphs<sup>1</sup>

The likelihood  $\mathcal{L}_{\mathcal{D}|\mathcal{G}}$  of an ancestral graph  $\mathcal{G}$  is decomposable in terms of multivariate cross-information  $I(\mathcal{C})$ , summed over all *ac*-connected subsets of variables  $\mathcal{C}$ :

$$\mathcal{L}_{\mathcal{D}|\mathcal{G}} = \frac{1}{Z_{\mathcal{D},\mathcal{G}}} \exp \left( N \sum_{\substack{\text{ac-connected} \\ \mathcal{C} \subseteq \mathcal{V}}} (-1)^{|\mathcal{C}|} I(\mathcal{C}) \right)$$

where  $N$  is the number of iid samples in dataset  $\mathcal{D}$ , and  $Z_{\mathcal{D},\mathcal{G}}$  is a data- and model-dependent normalization constant.

<sup>1</sup> Lagrange & Isambert, ICML 2025

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## Definition: ac-connected subset

A subset  $\mathcal{C}$  is said to be *ac*-connected if  $\forall X, Y \in \mathcal{C}$ , either:

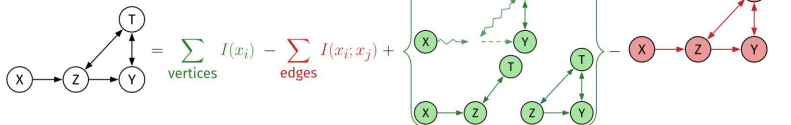
- $X$  and  $Y$  are connected through any type of edge, or
- there exists an **ac-connecting path** between  $X$  and  $Y$  given  $\mathcal{C}$ .

An **ac-connecting path** between  $X$  and  $Y$  given  $\mathcal{C}$  is a collider path

$$X^* \rightarrow Z_1 \leftrightarrow \dots \leftrightarrow Z_k \leftarrow *Y,$$

such that each  $Z_i \in \text{An}_{\mathcal{G}}(\{X, Y\} \cup \mathcal{C})$ , that is, with  $Z_i \in \mathcal{C}$  or through an ancestor path, i.e.,  $Z_i \rightarrow \dots \rightarrow T$  with  $T \in \{X, Y\} \cup \mathcal{C}$ .

## Cross-entropy decomposition of ancestral graph:



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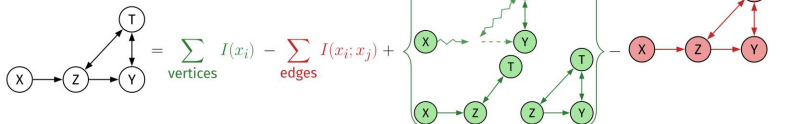
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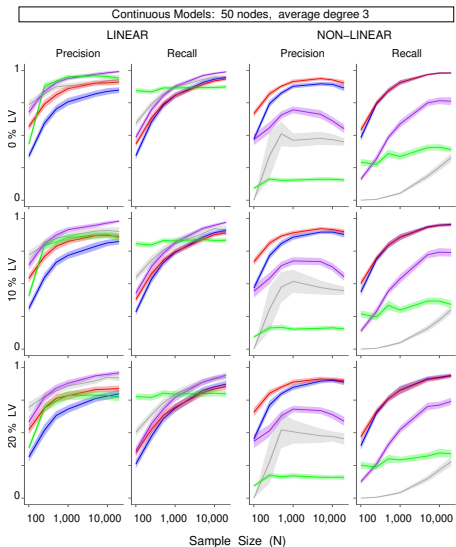


## Proposition: estimation of cross-information for *ac*-connected subset $C$

$$I(C) = \sum p(x, y, z) \log \frac{q(x, y, z) q(x) q(y) q(z)}{q(x, y) q(x, z) q(y, z)} \equiv \sum p(x, y, z) \log \frac{p(x, y, z) p(x) p(y) p(z)}{p(x, y) p(x, z) p(y, z)}$$

# Benchmarks on Linear vs Non-linear Continuous Data

MIIC does not assume linear distribution  $\Rightarrow$  MIIC outperforms other methods on non-linear data

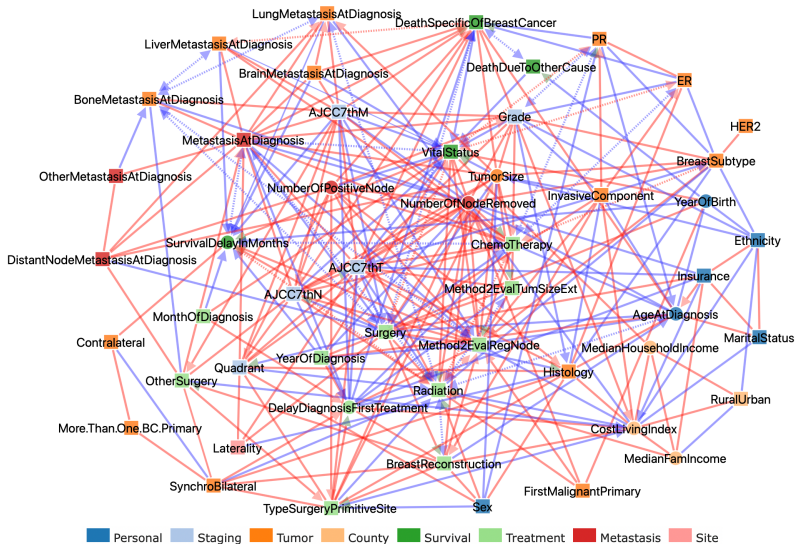


— M3HC — GFCI — DAG-GNN — MIIC — MIIC\_search&score

# Learning causal networks from nationwide healthcare data

400,000 breast cancer patients from SEER database

Ribeiro-Dantas et al. iScience 2024



⇒ **Reliable** causal discovery: **90%** of causal edges are **correct** (+ **8%** likely correct too)



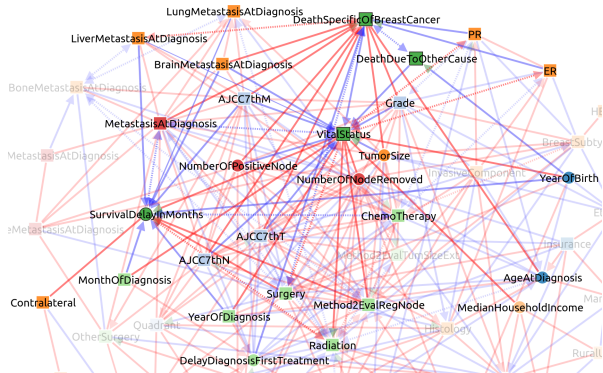
# 'Putative' *versus* 'Genuine' causal edges

'Putative' causal edge ( $p_t = p_h = 0.5 \rightarrow p_h > 0.5$ ) *versus* 'Genuine' causal edge ( $p_t > 0.5 \rightarrow p_h > 0.5$ )

ie Putative  $\rightarrow$   $\equiv$  Genuine  $\rightarrow$  **or** latent common cause  $\leftrightarrow$

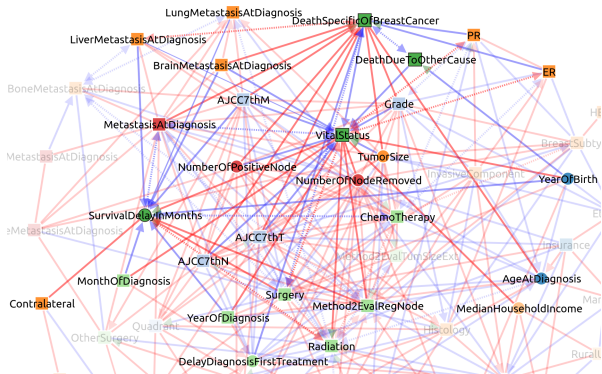


# SEER Database: Survival subnetwork





# SEER Database: Survival subnetwork



- Expected causal edges:
- Metastasis at diagnosis → Death specific of breast cancer
  - ER → Death specific of breast cancer
  - Year of birth → Death due to other cause
  - Year of birth → Vital status
  - Year of birth → Survival delay in months due to 2010-2016 censoring!
  - Vital status → Survival delay in months

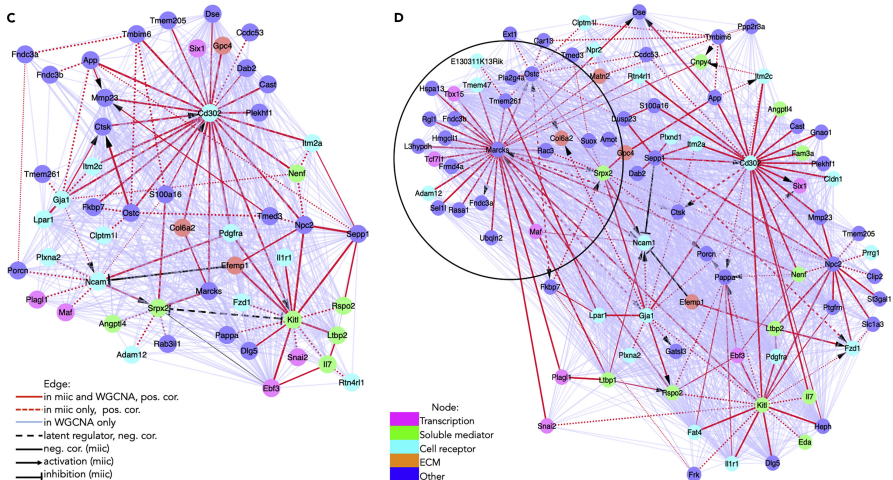
Unexpected missing edges: Insurance ... any Survival variable! 70% explained through treatment variables

# Learning Gene Regulatory Network from scRNAseq data

Gene networks in bone marrow hematopoietic stem cell-supporting stromal niche population

P Chabord, Sorbonne Université (scRNAseq data from Tikhonova *et al.* Nature 2019)

Desterke *et al.* iScience 2020

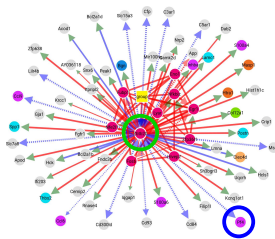
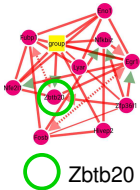


Gene regulatory networks of Leptin receptor-positive cell population at baseline (C, 1,712 cells) and under stress (D, 3,467 cells)

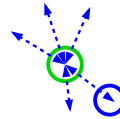
# Extension to VERY LARGE Gene Regulatory Networks

scRNAseq data: H el ene Moreau, Institut Curie

10 TFs + 205 non-TFs



Latent variable



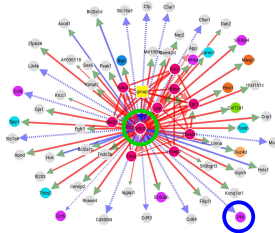
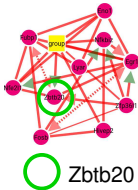
⇒ TF-MIIC can scale to hundreds TFs and thousands non-TFs

Fr ank Simon

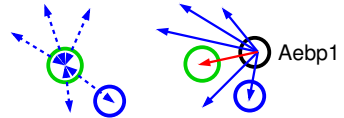
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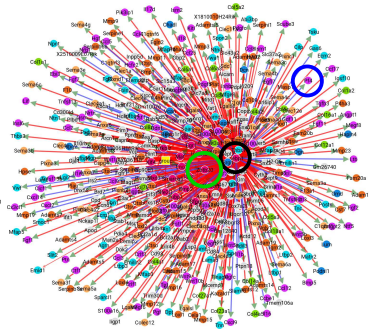
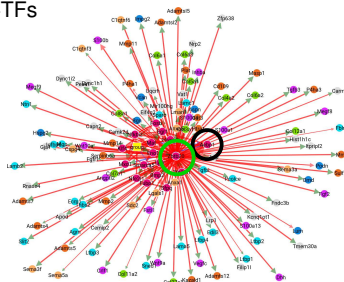
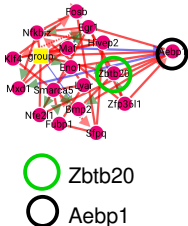
10 TFs + 205 non-TFs



Latent variable



17 TFs + 703 non-TFs

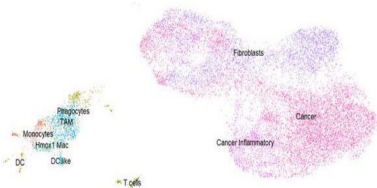
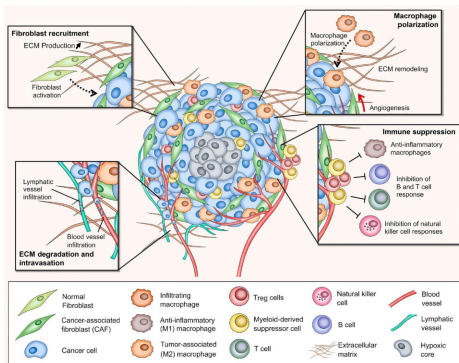


⇒ TF-MIIC can scale to hundreds TFs and thousands non-TFs

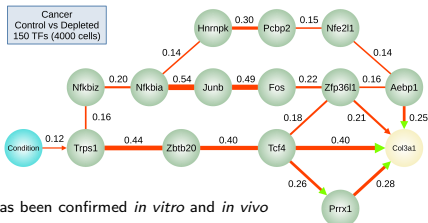
Franck Simon

# Dissecting complex biological responses in scRNAseq data

Macrophages prevent T cell infiltration in tumors by controlling ECM remodeling by cancer cells



Data from: Zoé Fusilier & Hélène Moreau (Institut Curie, U932)  
 Analysis: Franck Simon (Isambert's team)



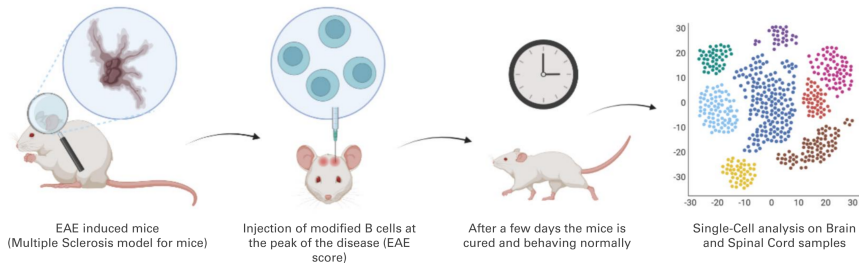
Tcf4 → Col3a1 has been confirmed *in vitro* and *in vivo*

Fusilier *et al.* Science Immunology 2026

# Learning Gene Network Response to Cellular Therapy

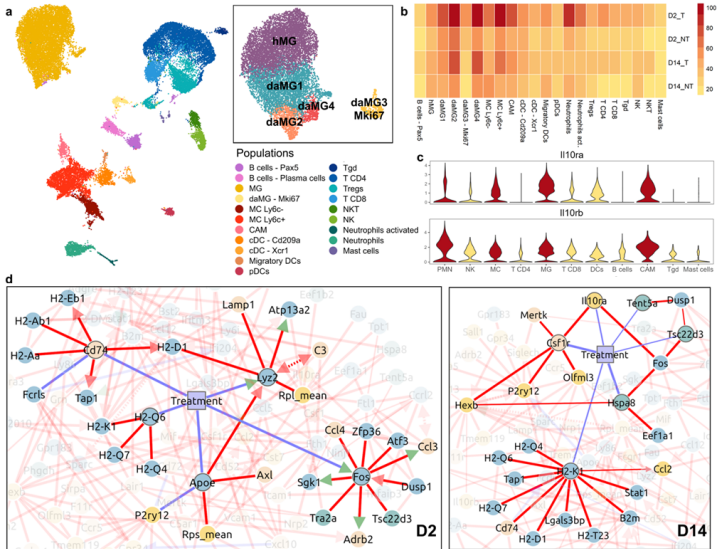
Analysis of scRNAseq data from Fillatreau Lab, Institut Necker Enfants Malades

## Cellular therapy against Multiple Sclerosis (injection of IL-10 producing Bregs):



Analysis of scRNAseq data: Louise Dupuis

# Learning Gene Network Response to Cellular Therapy



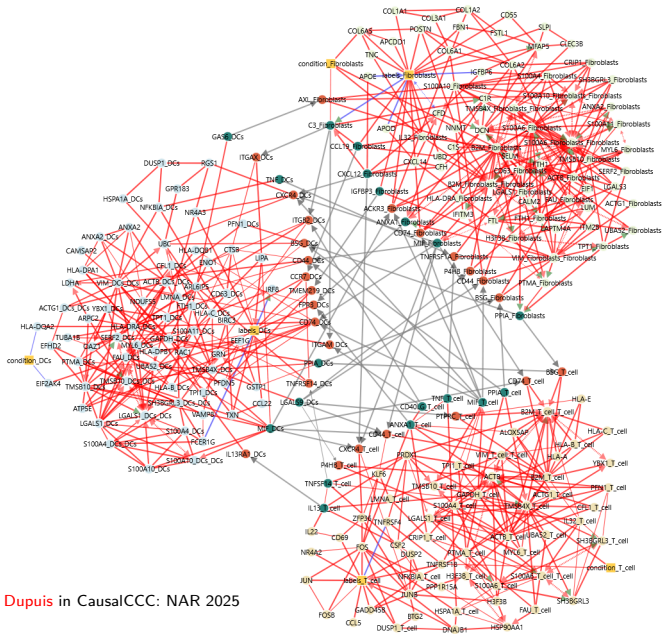
Manfroi *et al.* Induced regulatory B cells stably expressing IL-10 cure CNS autoimmunity by targeting microglia. *under revision* (2024)

Analysis of scRNAseq data  
Louise Dupuis





# Extension to CCC Pathways across Multiple Cell Types



Louise Dupuis in CausalCCC: NAR 2025

# CausalCCC Pathways from Spatial Transcriptomics Data

## Myocardial Infarction (ST data: Kuppe *et al.* Nature 2022)

A

**Original study: Spatial multi-omic map of human myocardial infarction (Kuppe *et al.*)**



31 samples from 23 patients

snRNAseq: 191,795 cells

snATACseq: 46,086 cells

26 Spatial Transcriptomics (10X)

**Subset: Patient P9**  
10X Visium slide



Input: n= 4,084 cells in ischemic zone



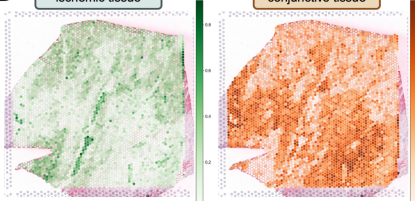
**CausalCCC default pipeline**

80 sender nodes, 80 receiver nodes, 17 L-R links, SPP1 as gene of interest

B

Myocardiac ischemic tissue

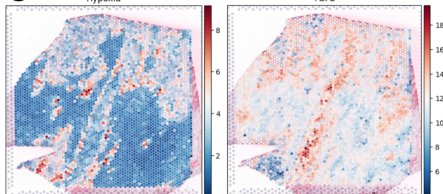
Cardiac conjunctive tissue



C

Hypoxia

TGFb



⇒ **Cell-Cell Communication** between:

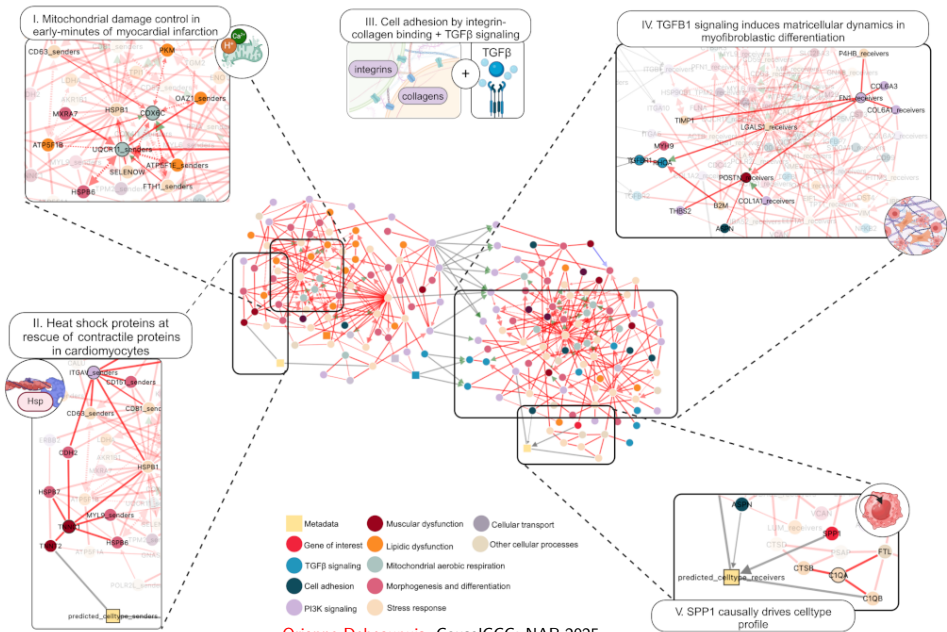
ischemic myocardial **greens** (including cardiomyocytes, vascular smooth muscle cells, endothelial cells, pericytes) and **connective tissue** (including fibroblasts and myeloid cells)

Orianne Debeauvais

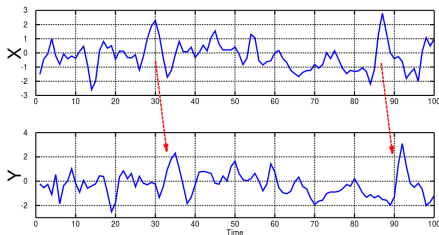
CausalCCC: NAR 2025



# CausalCCC Pathways from Spatial Transcriptomics Data



# Extension to Time Series Data $X_{t' < t} \rightarrow Y_t$

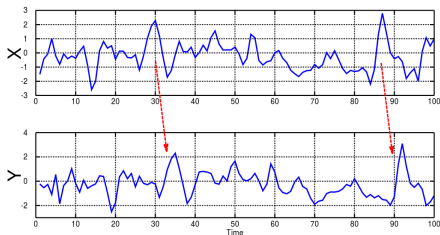


## Granger Causality (Granger *Econometrica* 1969)

→ Best regression of  $Y_t$  with or without lagged values of  $X_{t' < t}$  ?

$$Y_t = a_0 + a_1 Y_{t-1} + a_2 Y_{t-2} + \cdots + a_m Y_{t-m} + b_p X_{t-p} + \cdots + b_q X_{t-q} + \epsilon_t$$

# Extension to Time Series Data $X_{t' < t} \rightarrow Y_t$



## Granger Causality (Granger *Econometrica* 1969)

→ Best regression of  $Y_t$  with or without lagged values of  $X_{t' < t}$  ?

$$Y_t = a_0 + a_1 Y_{t-1} + a_2 Y_{t-2} + \dots + a_m Y_{t-m} + b_p X_{t-p} + \dots + b_q X_{t-q} + \epsilon_t$$

## Transfer Entropy (Schreiber *PRL* 2000)

→  $T_{X \rightarrow Y} = I(Y_t; \mathbf{X}_{t' < t} | \mathbf{Y}_{t' < t})$  versus  $T_{Y \rightarrow X} = I(X_t; \mathbf{Y}_{t' < t} | \mathbf{X}_{t' < t})$

Then, if  $T_{X \rightarrow Y} > 0$  &  $T_{Y \rightarrow X} \simeq 0 \implies X_{t' < t} \rightarrow Y_t$

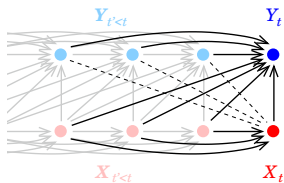
Although always the possibility of a **latent variable**:  $X_{t'} \leftarrow L_{t''} \rightarrow Y_t$ , with  $t'' < t' < t$

Temporal Causality  $\leftrightarrow$  Graph-based Causality ??

# Temporal Causality $\leftrightarrow$ Graph-based Causality ??

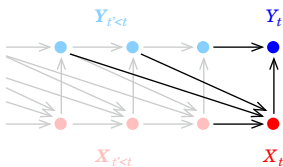
IDEA: **Information-based reasoning on time-unfolded graphs  $\mathcal{G}_t$**

A



$$I(X_t; Y_{t-1} | X_{t-1}) = 0 \implies \begin{array}{c} Y_{t-1} \rightarrow Y_t \\ \uparrow \\ X_t \end{array}$$

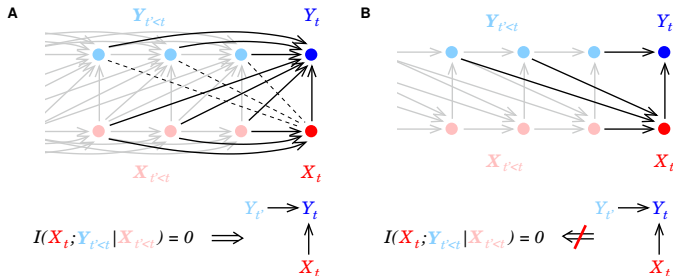
B



$$I(X_t; Y_{t-1} | X_{t-1}) = 0 \not\Leftarrow \begin{array}{c} Y_{t-1} \rightarrow Y_t \\ \uparrow \\ X_t \end{array}$$

# Temporal Causality $\iff$ Graph-based Causality ??

IDEA: **Information-based reasoning on time-unfolded graphs  $\mathcal{G}_t$**



**Theorem 1** [ Temporal Causality  $\implies$  Time-unfolded Graph-based Causality ]

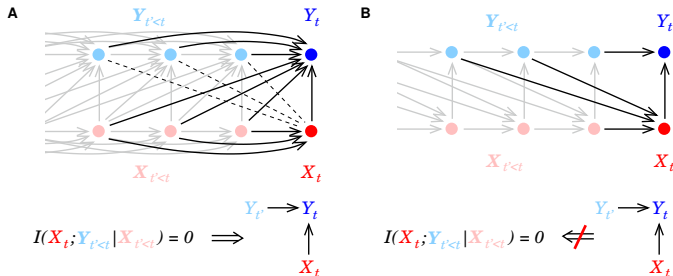
If  $Y_t - X_t$  &  $T_{Y \rightarrow X} = I(X_t; Y_{t' < t} | X_{t' < t}, \mathbf{V}_{t' \leq t}^{X, Y}) = 0$

Then  $\forall Y_{t'} - Y_t, Y_{t'} \rightarrow Y_t \leftarrow X_t$  [ &  $\forall t' < t, Y_{t'} \not\rightarrow X_t$  ]

But **the converse** is not true !

# Temporal Causality $\Leftrightarrow$ Graph-based Causality ??

IDEA: **Information-based reasoning on time-unfolded graphs  $\mathcal{G}_t$**



**Theorem 1** [ Temporal Causality  $\implies$  Time-unfolded Graph-based Causality ]

If  $Y_t - X_t$  &  $T_{Y \rightarrow X} = I(X_t; Y_{t' < t} | X_{t' < t}, \mathbf{V}_{t' \leq t}^{X, Y}) = 0$

Then  $\forall Y_{t'} - Y_t, Y_{t'} \rightarrow Y_t \leftarrow X_t$  [ &  $\forall t' < t, Y_{t'} \not\rightarrow X_t$  ]

But **the converse** is not true !

F Simon et al., eLife 2025

Hence, Granger-Schreiber **temporal causality** is **too restrictive** and may **miss actual causal relations** uncovered by **graph-based causal discovery** methods for time series

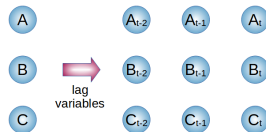
# tMIIC: Causal discovery from time series data

Preprocessing

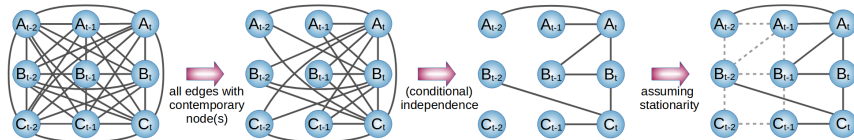
	A	B	C
1			
2			
3			
4			
5			
6			
7			

lag data

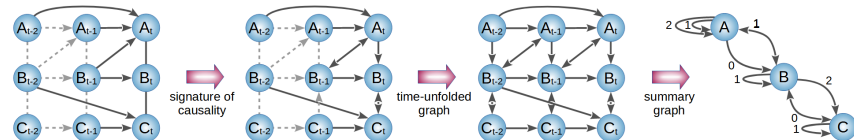
	$A_{t-2}$	$B_{t-2}$	$C_{t-2}$	$A_{t-1}$	$B_{t-1}$	$C_{t-1}$	$A_t$	$B_t$	$C_t$
3									
4									
5									
6									
7									



Skeleton



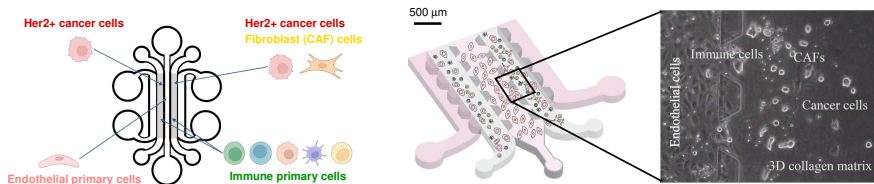
Orientation



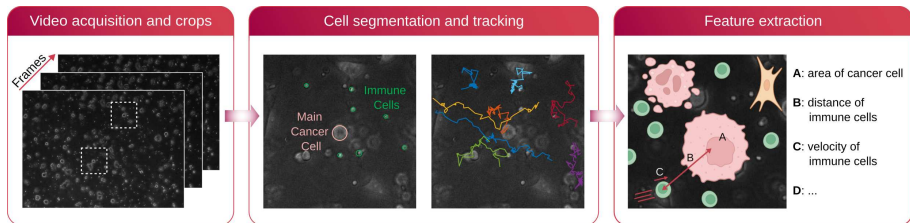
# CausalXtract: Causal discovery from live cell images

## Analysing ex-vivo tumor-immune response ecosystems

### a Tumor-on-chip preparation



### b CausalXtract's live-cell image feature extraction module (CellHunter+)



# CausalXtract: Causal discovery from live cell images

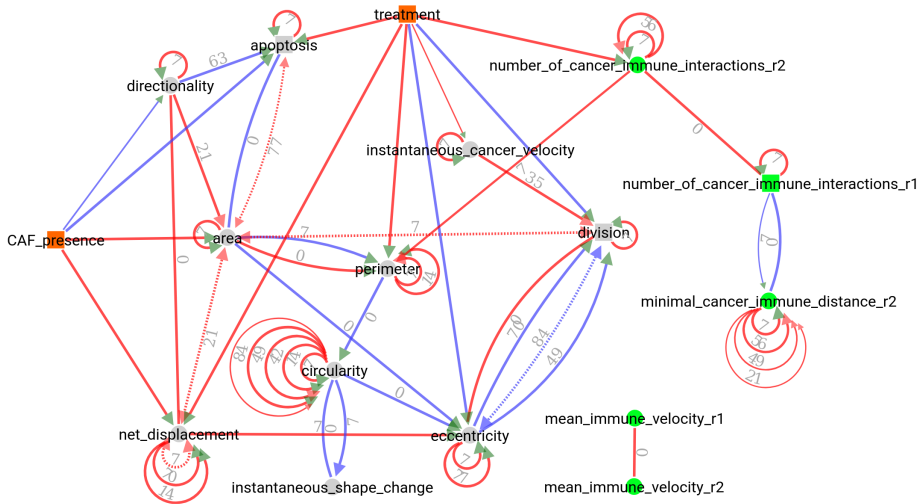
From the experiments, we get a dataset with 21 variables about:

- Experimental conditions (CAF presence and Treatment)
- Cell Shape
- Cell Motility
- Cell division
- Cell apoptosis
- Cell Interactions with immune cells
- Cell Motility of immune interacting cells

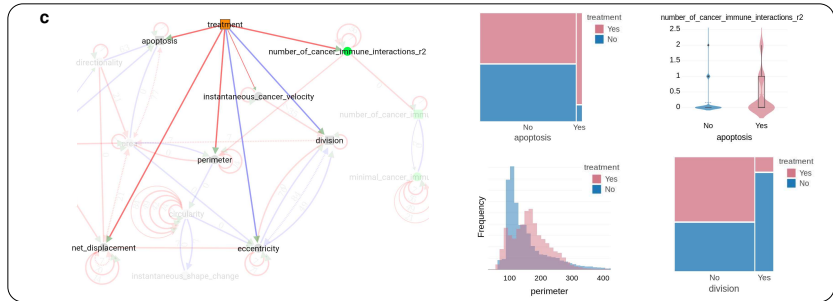
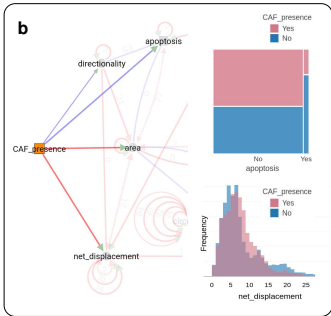
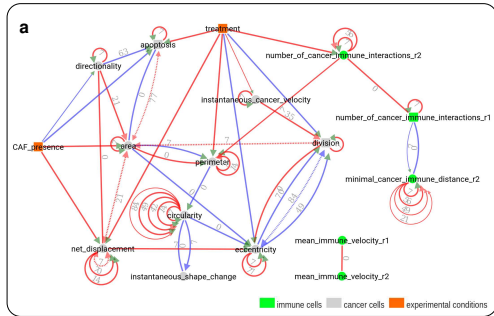
IDFrame	CAFPresence	Treatment	CAF	Apoptosis	OC	Duplicator	Area	InstSpeed	InstChange	Shape	Perimeter	Eccentricity	Circularity	TotDist	NetDist	Straightness	l	min	dir	c	mCellsBehind	mCellsInFront	cum	a	e	at	b	Apoc	IDCell
1	1	0	0	0	0	0	2759NA	NA	206.326	0.5033495	0.81443NA	NA	NA	NA	NA	1	0	NA	NA	NA	NA	NA	1	0	0	0	0	1	
2	1	0	0	0	0	0	27340.484055429697037	0.2691388420211	206.967	0.4768021	0.801904	0.4841	0.4846	1	2	62.648622	1	1	1	0	0	1	1	0	0	0	1		
3	1	0	0	0	0	0	27440.237006202304126	0.1078025996685	210.538	0.4644681	0.777931	0.72110	0.26112	0.362132162	1	70.178345	1	1	2	0	0	1	1	0	0	0	1		
4	1	0	0	0	0	0	26899	1.20343470673425	0.5953720808029	206.278	0.4787802	0.794151	1.9245	1.15851	0.601982474	3	68.154236	2	1	3	0	0	1	1	0	0	1		
5	1	0	0	0	0	0	26420.727902948856354	0.5136151313782	205.61501	0.4938549	0.785295	2.6524	1.87705	0.707678437	1	67.208633	1	2	4	0	0	1	1	0	0	0	1		
6	1	0	0	0	0	0	25950.513986945152289	0.5182042717934	203.951	0.4921426	0.783963	3.1654	1.36433	0.430978699	2	67.082039	2	1	5	0	0	1	1	0	0	0	1		
7	1	0	0	0	0	0	26000.780381560325263	0.0953499640485	216.235	0.4587875	0.698178	3.94682	1.2051	0.53727746	2	63.786876	2	1	6	0	0	1	1	0	0	0	1		
8	1	0	0	0	0	0	2525	1.26677143573761	0.8389224796295	209.123	0.4476641	0.725564	5.2135	3.36114	0.644694746	3	62.225395	2	1	7	0	0	1	1	0	0	1		
9	1	0	0	0	0	0	25610.767565548419953	0.4027699828148	198.617	0.4676091	0.815805	5.98112	6.2086	0.438189387	2	67.941154	3	0	8	0	0	1	1	0	0	0	1		
10	1	0	0	0	0	0	2539	1.08244478702545	0.2457981854677	199.651	0.4928071	0.800442	7.06352	8.16669	0.398763925	2	64.84597	2	1	9	0	0	1	1	0	0	1		
11	1	0	0	0	0	0	2448	0.97378009597724	1.01680935030395	206.433	0.5309519	0.722172	8.0373	8.05104	0.454260737	2	63.953106	2	1	10	0	0	1	1	0	0	1		
12	1	0	0	0	0	0	24240.657828079239077	0.235748054518	220.674	0.4987783	0.625519	8.6949	4.2981	0.494326532	1	65.946945	1	2	11	0	0	1	1	0	0	0	1		
13	1	0	0	0	0	0	25180.664140850902411	1.0689306516647	195.29201	0.4905526	0.824653	9.3594	0.00901	0.428357989	1	65.11528	1	2	12	0	0	1	1	0	0	0	1		
14	1	0	0	0	0	0	25620.856808543205261	0.4925665855408	201.586	0.4675879	0.782261	10.2164	7.0335	0.460399002	0	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	
15	1	0	0	0	0	0	25350.438889145851135	0.301750510931	202.397	0.5125131	0.777842	10.6554	3.1183	0.40469721	1	63.702435	1	2	13	0	0	1	1	0	0	0	1		
16	1	0	0	0	0	0	2552	1.03880643844605	0.190177321434	198.48	0.5062683	0.830872	11.6943	4.66985	0.296732813	2	57.939625	2	1	14	0	0	1	1	0	0	1		
17	1	0	0	0	0	0	25860.353826367185593	0.4893024861813	202.876	0.506636	0.781818	12.0475	5.1972	0.292162091	1	57.564721	2	1	15	0	0	1	1	0	0	0	1		
18	1	0	0	0	0	0	26080.759838441562653	0.1327250599861	198.81599	0.4908354	0.829115	12.806	3.40004	0.265507966	1	65.11528	1	2	16	0	0	1	1	0	0	0	1		
19	1	0	0	0	0	0	25430.754866003990173	0.7226305007935	195.946	0.4822299	0.832307	13.5613	1.9503	0.235610142	1	64.381676	1	2	17	0	0	1	1	0	0	0	1		
20	1	0	0	0	0	0	25400.3022449612611749	0.0335738621652	194.358	0.4747291	0.844965	13.8632	8.9724	0.208992615	1	65.795135	1	2	18	0	0	1	1	0	0	0	1		

The dataset contains 36 trajectories including upto 1440 time-lapse images ( $\delta\tau = 2\text{min}$ ), corresponding to a total of 46,935 images.  $\rightarrow$  avg relaxation time  $25\delta\tau$ : 12 layers of  $5\delta\tau$

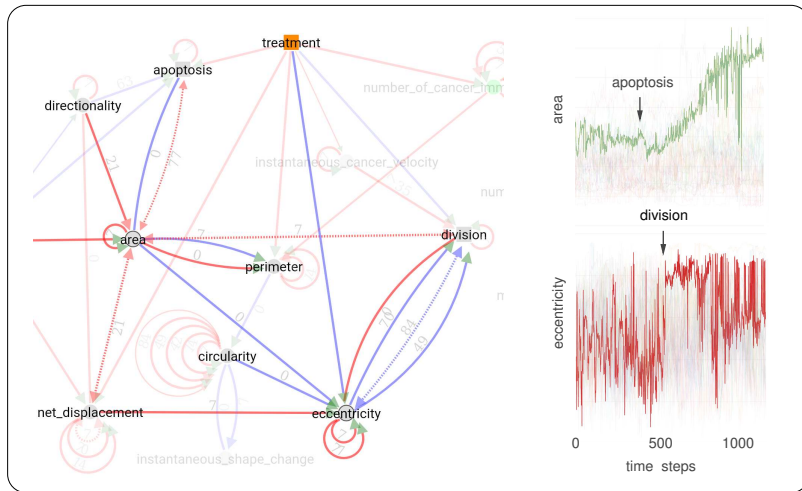
# CausalXtract: Causal discovery from live cell images



# CausalXtract: Causal discovery from live cell images



# CausalXtract: Opposite effects at different time lags



Ali Chemkhi Alexander Chervov Louis Montagne Ouahab Elkouadi  
 Tiziana Tocci Hugo Mousset Franck Simon Louise Dupuis Orianne Debeaupuis



Gael Simon Vincent Cabell Nadir Sella Severine Affeldt Param Priya Singh



Nikita Lagrange Marcel Ribeiro Dantas Honghao Li Louis Verny Julia Malagutti

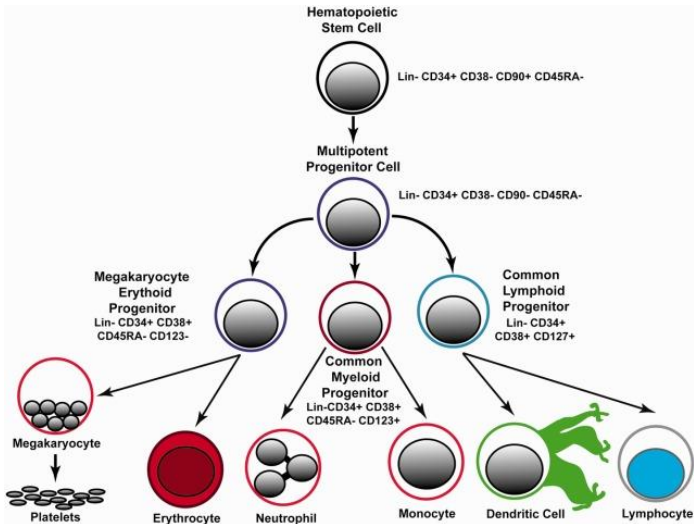
# The Team (Lab retreat, Luminy, Marseille, Sept 2023)



Franck, Gaël, Hervé, Nikita, Louis, Louise, Tiziana, Oriane, Liza

Coll: Fillatreau (INEM), Charbord (SU), Bravi (ICL), Parrini (U830), Perie, Hersen (UMR168), Hamy, Moreau, Lennon, Salmon (U932)

# Application to Human Hematopoietic Stem Cells



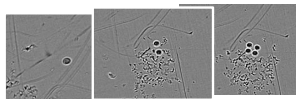
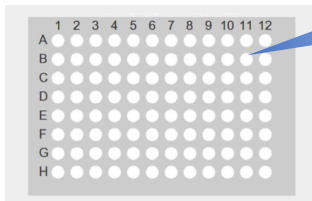
Cimato et al., 2015

collab. with Perié lab, Institut Curie

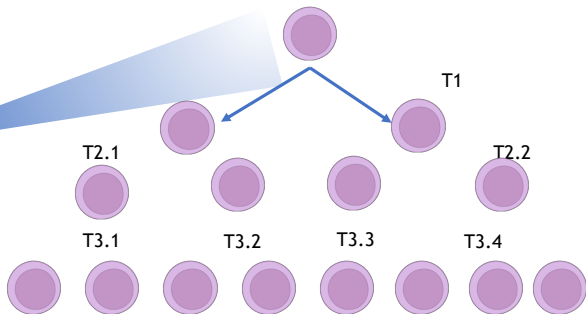
# Application to Human Hematopoietic Stem Cells

## Data collection

Acquisition with Incucyte



Cell segmentation project



Frames from  $t_0$  to  $t=90$  hours

**Sequencing at 96 hours (SMART-SEQ3)**

scRNASeq project

## Cell counting

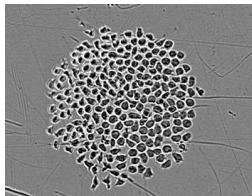
### Two key properties:

- synchronicity in **division**
- similarity in fate

among progenies of individual stem and progenitor cells

### Current goal:



- automatise cell counting after the first **mitosis** (= segmentation)

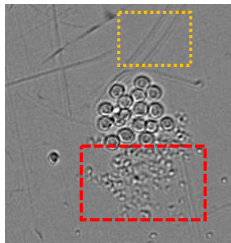


In collaboration with **Eugenio Martinelli**, **Arianna Mencattini** University of Rome Tor Vergata)

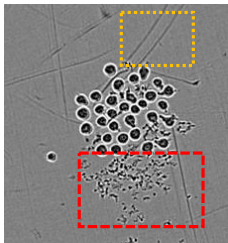
# Application to Human Hematopoietic Stem Cells

## Issues

- Out of focus frames
- Small cells
- Debris 
- Plastic defects 



Out of focus



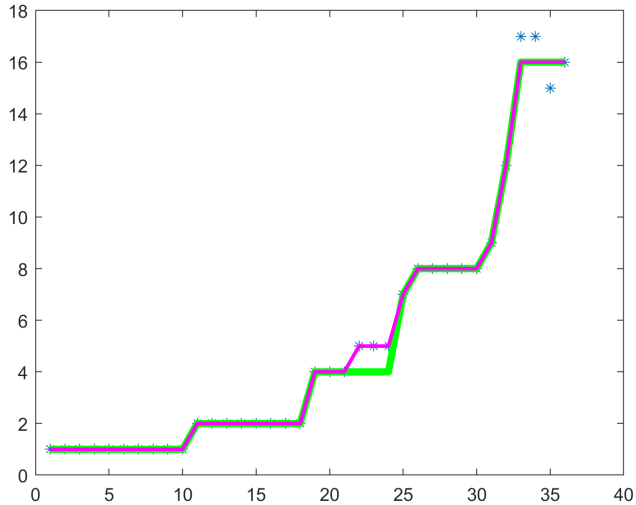
In focus



Deep Learning could handle better  
these issues

# Application to Human Hematopoietic Stem Cells

Stepwise correction



## scRNASeq data

### Raw matrixes

- Dataset1: 70 cells, 28750 genes
- Dataset2: 72 cells, 30682 genes

### Metadata

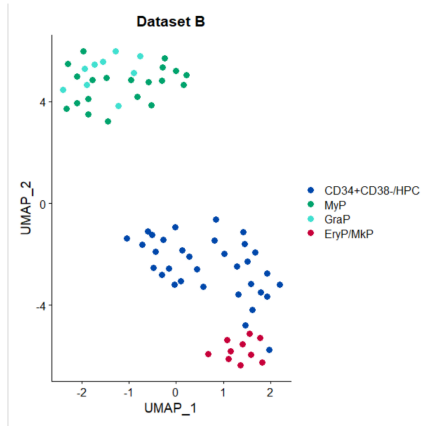
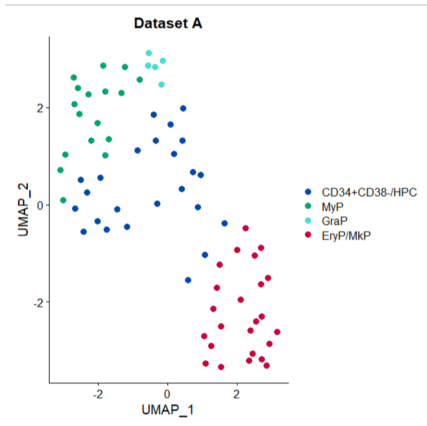
- Family (or well) - 30 families
- Times of division: T1, T2.i, T3.j
- Colony size (at 96 hours)
- Fate/predicted cell type

Sequencing at 96 hours

Cell type: MPP

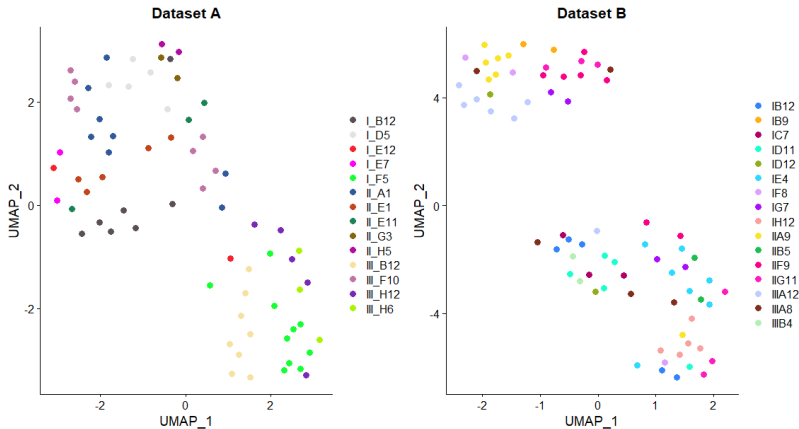
# Application to Human Hematopoietic Stem Cells

## Cell annotation



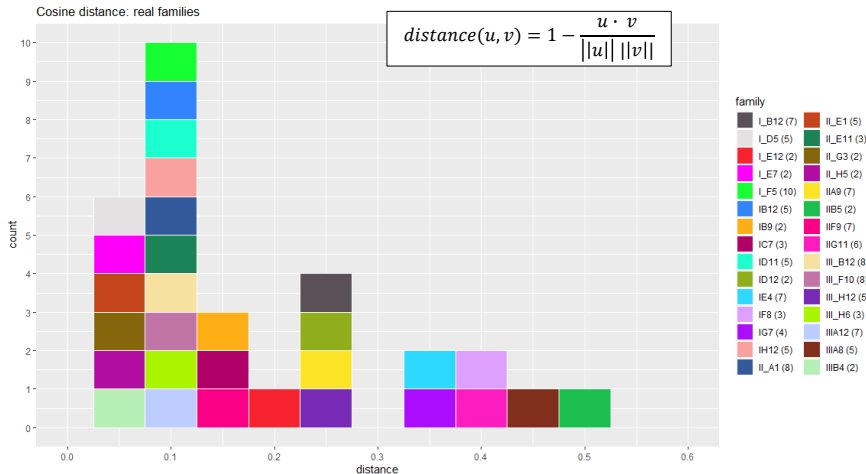
# Application to Human Hematopoietic Stem Cells

## Families tend to cluster together

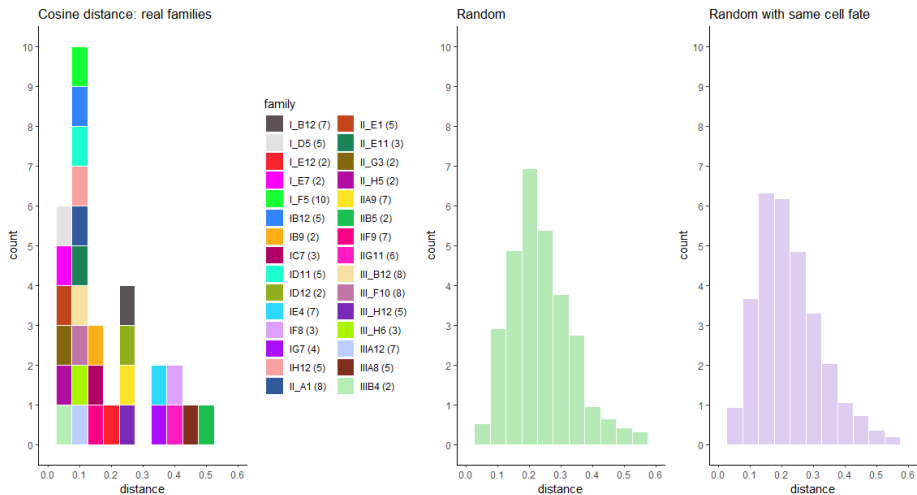


# Application to Human Hematopoietic Stem Cells

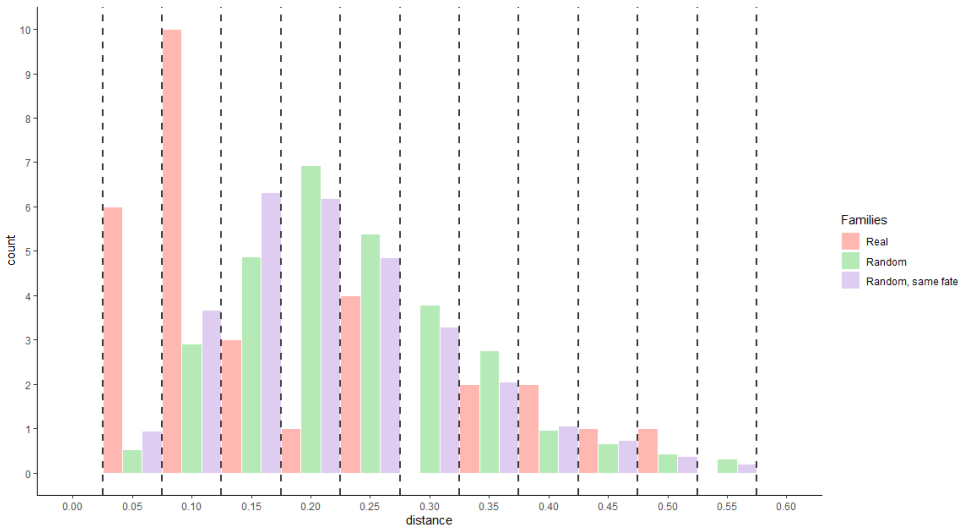
Cosine distance



# Application to Human Hematopoietic Stem Cells



# Application to Human Hematopoietic Stem Cells

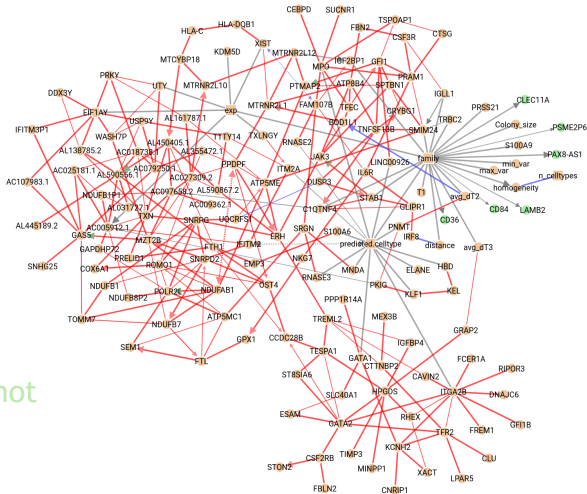


# Application to Human Hematopoietic Stem Cells

## TF-MIIC

Franck Simon

- Regular MIIC: up to 500 variables
- TF (regulators)-MIIC: a few thousands of variables
- Variables: regulators or not regulators



collab. with Perié lab, Institut Curie

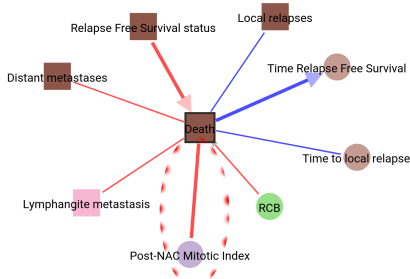
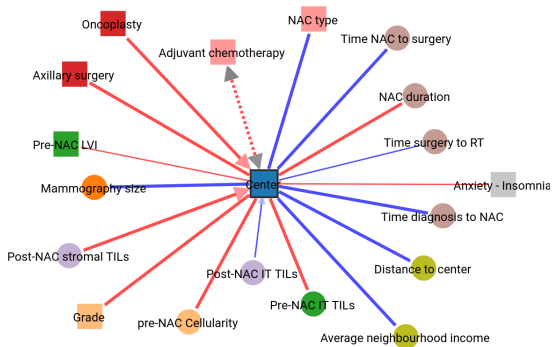




# MIIC finds unexpected biases and new prognostic markers

NEOREP cohort: 1,200 patients, Reyal lab, Curie Hospitals.

Sella *et al.* npj Digital Medicine 2022



Center bias (Paris / St Cloud hospitals):

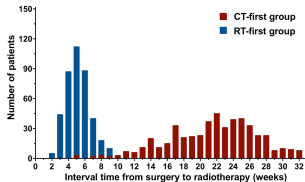
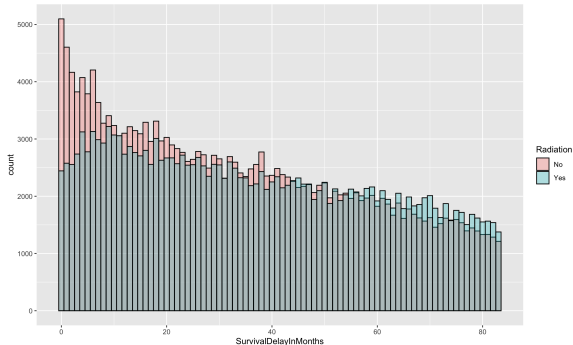
- Different clinical profiles
- Different clinical practices

Post-NAC Mitotic Index:

- New prognostic markers
- in addition to RCB

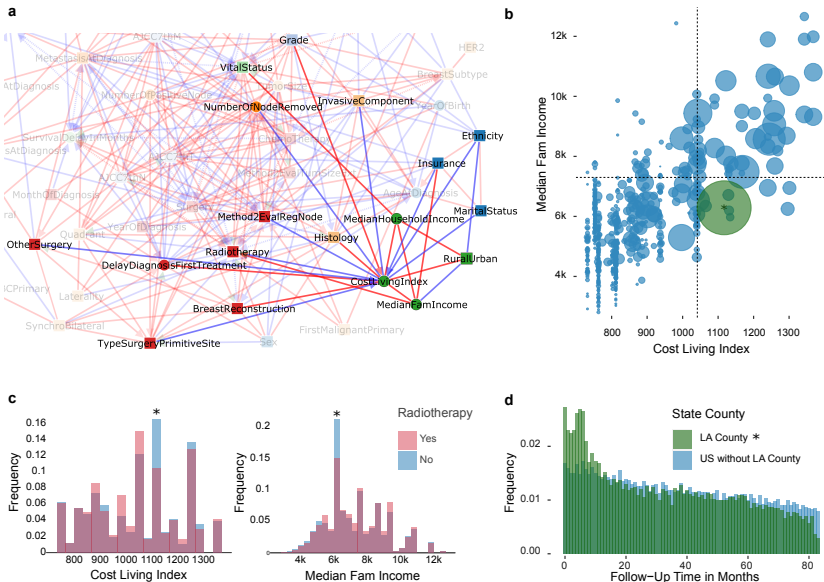
# SEER Database: Survival subnetwork

Interpretation of causal edge: Vital Status  $\rightarrow$  Radiotherapy in terms of  $\approx$  6-7 month delays





# SEER Database: Socio-economic subnetwork



\* L.A. county (10% of dataset): **32.6% Radiotherapy** in L.A. vs 50% rest of US; **7% of L.A. patients drop out** vs 1.5% rest of US

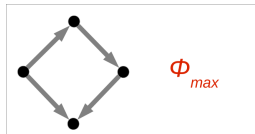
# How to Learn Graphical Models from Observational Data

*a one-slide primer*

## 1. Search & Score ('Bayesian', scoring function $\phi$ )

Find  $\mathcal{G}$  that **maximizes** Likelihood  $\phi_{\mathcal{G}} = \mathcal{L}_{\mathcal{D}|\mathcal{G}}$

**Super-exponential space** of networks, **only**  $\rightarrow$



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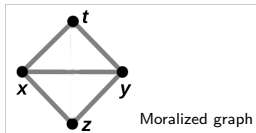
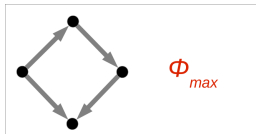
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Compute  $L = D - A \sim \Sigma^{-1}$  (assuming linearity)

**Invertibility/Stability issue** and **only** — edges



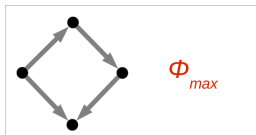
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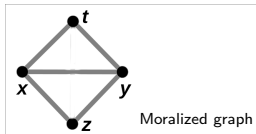
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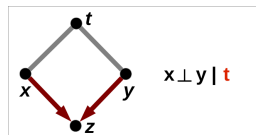
**Invertibility/Stability issue** and **only** — edges



## 3. Constraint-based (Conditional independences)

**Broader network class** including  $\rightarrow \leftrightarrow$

**Slow** and **Not robust to sampling noise**



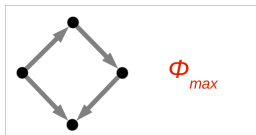
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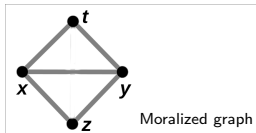
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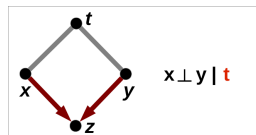
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**MIIC: a robust Network Inference method combining constraints and scores**

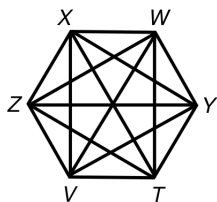
Affeldt et al. 2015; Verny et al. 2017; Cabeli et al. 2020; Ribeiro-Dantas et al. 2024; Simon et al. 2025; Lagrange et al. 2025

# MIIC combines Information theory with Causal discovery

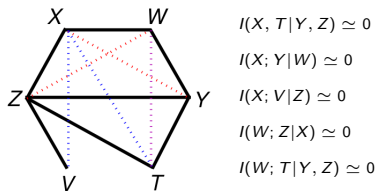
MIIC first **abstracts** iteratively the **most significant contributions** to each  $I(X, Y)$

$$I(X; Y) - I(X; Y; A_1) - I(X; Y; A_2|A_1) - \dots - I(X; Y; A_n|\{A_i\}_{i=1}^{n-1}) = I(X; Y|\{A_i\})$$

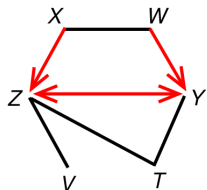
(0) Complete graph



(1) Remove edges  $I(X; Y|\{A_i\}) \simeq 0$

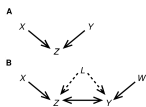


(2) Orient V-structures ( $p_{\text{head}}$ )

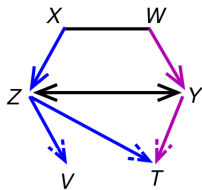


$$I(X; Y; Z|W) < 0$$

$$I(W; Z; Y|X) < 0$$



(3) Propagate orientations ( $p_{\text{tail}}$ )



$$I(X; V; Z) > 0$$

$$I(X; T; Z|Y) > 0$$

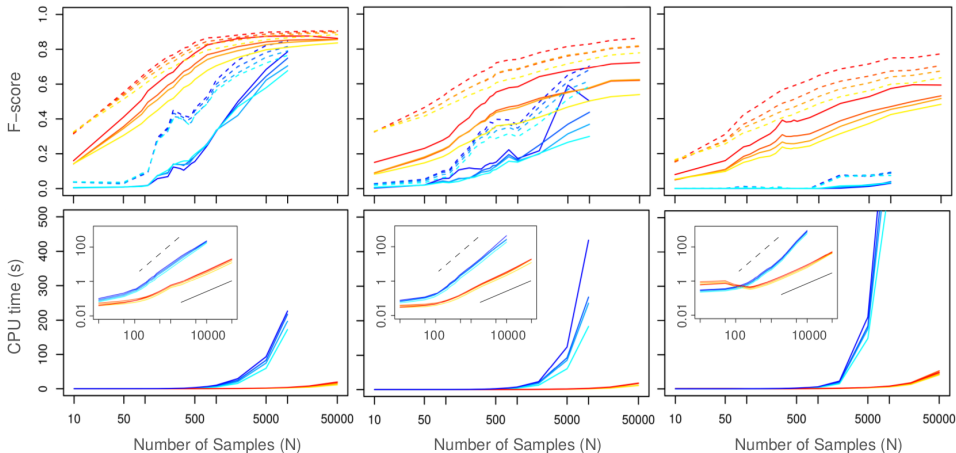
$$I(W; T; Y|Z) > 0$$

# Benchmark on Ancestral Graphs (including $\rightarrow$ $\leftrightarrow$ edges)

Alarm: 37 nodes, 46 links, 509 param, Avg deg 2.49

Insurance: 27 nodes, 52 links, 984 param, Avg deg 3.85

Barley: 48 nodes, 84 links, 114,005 param, Avg deg 3.5



--- Graph skeleton  
 — Graph incl. orientations

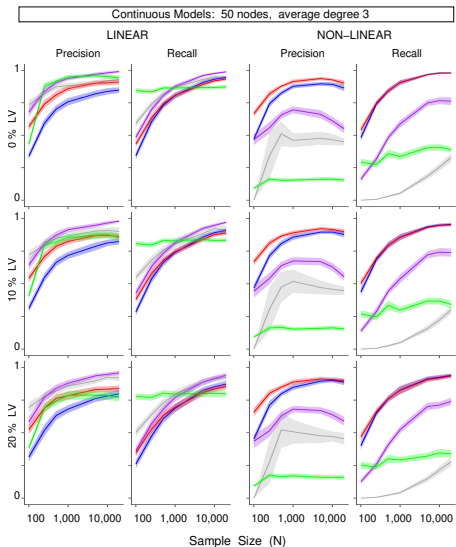
— miic : 0 latent variables  
 — miic : 5 latent variables  
 — miic : 10 latent variables  
 — miic : 20 latent variables  
 — RFCI : 0 latent variables  
 — RFCI : 5 latent variables  
 — RFCI : 10 latent variables  
 — RFCI : 20 latent variables

better/comparable results w.r.t RFCI  
 with 100× fewer samples  
 and 100× faster

Verny et al. PLoS Comput Biol 2017

# Benchmarks on Linear vs Non-linear Continuous Data

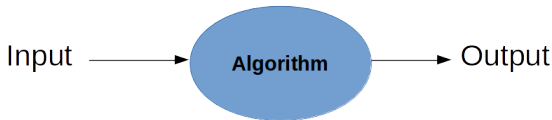
MIIC does not assume linear distribution  $\Rightarrow$  MIIC outperforms other methods on non-linear data



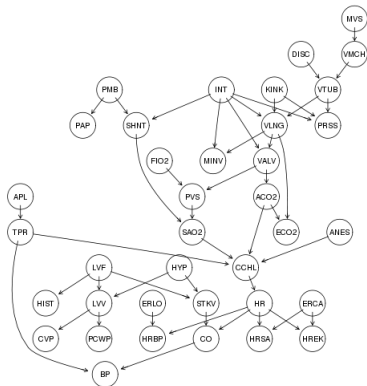
— M3HC — GFCI — DAG-GNN — MIIC — MIIC\_search&score

# What is network reconstruction ?

Disentangling **direct** from **indirect** relations between variables, including *or* excluding **cause-effect** relationships.



	CVP	PCWP	HIST	TPR
1	0.175098608713597	0.288900742307305	0.55428269	0.857808550121263
2	0.0933099433314055	0.218888618983328	0.47473369	0.332442865008488
3	0.690925023518503	0.861214176984504	0.25389608	0.849817770067602
4	0.572590821655467	0.549840433290228	0.15323819	0.715732422890142
5	0.857235474744812	0.255593683104962	0.49391256	0.37724070623517
6	0.590208335081115	0.367558936588466	0.62587462	0.933418722823262
7	0.816242689266801	0.526696094544604	0.47205955	0.651990963146091
8	0.76507281861268	0.835657971445471	0.96377760	0.984965795883909
9	0.885613681515679	0.196845271624625	0.50106454	0.293295677984133
10	0.941809490323067	0.956555964192376	0.07710378	0.941999231465161
11	0.685077040921897	0.517504557501525	0.49092264	0.731579512590542
12	0.0605227875057608	0.759360220748931	0.69840481	0.663990918546915
13	0.19431169051677	0.477279279148206	0.67160601	0.996502364054322
14	0.614625208079815	0.360529601573944	0.02014737	0.375805815681815
15	0.700897089438513	0.777111812029034	0.56314651	0.849968496710062

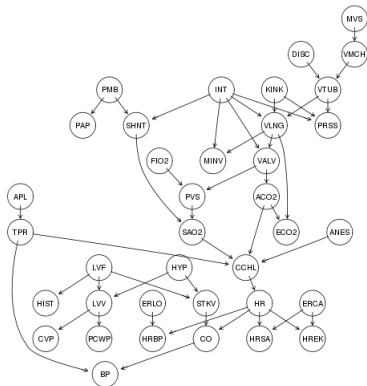


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	CVP	PCWP	HIST	TPR
1	2	0.288900742307305	A	0.857808550121263
2	3	0.218888618983328	C	0.332442865008488
3	5	0.861214176984504	B	0.849817770067602
4	1	0.549840433290228	C	0.715732422890142
5	17	0.255593683104962	A	0.37724070623517
6	9	0.367558936588466	A	0.933418722823262
7	7	0.526696094544604	B	0.651990963146091
8	1	0.835657971445471	B	0.984965795883909
9	3	0.196845271624625	C	0.293295677984133
10	5	0.956555964192376	A	0.941999231465161
11	1	0.517504557501525	D	0.731579512590542
12	13	0.759360220748931	C	0.663990918546915
13	9	0.477279279148206	B	0.996502364054322
14	7	0.360529601573944	A	0.375805815681815
15	1	0.777111812029034	B	0.849968496710062



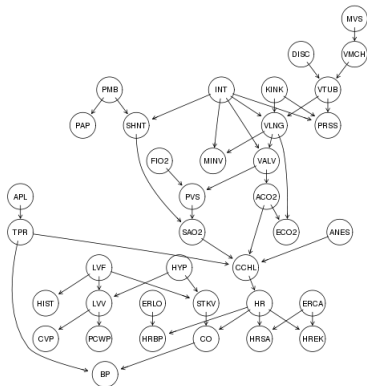


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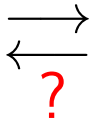
	CVP	PCWP	HIST	TPR
1	2	0.288900742307305	A	CASSLYNWLLDE
2		0.218888618983328	C	
3	5		B	TASSLYNWLLDR
4	1	0.549840433290228		CASSLYNWLLDE
5	17	0.255593683104962	A	CASSYGGNWLLDY
6	9	0.367558936588466	A	TASSLYNWLLDR
7		0.526696094544604	B	CASSLYNWLLDE
8	1	0.835657971445471	B	CASSYGGNWLLDY
9	3		C	
10	5	0.956555964192376		CASSLYNWLLDE
11	1		D	CASSYGGNWLLDY
12		0.759360220748931	C	TASSLYNWLLDR
13		0.477279279148206	B	CASSLYNWLLDE
14	7	0.360529601573944	A	
15	1	0.777111812029034	B	TASSLYNWLLDR



Can one Infer Causation from mere Correlation ?

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**Two-point** Correlation does not imply causation



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**Three-or-more-point** Correlation *might* imply causation

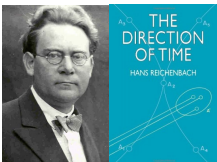
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**Three-or-more-point** Correlation *might* imply causation

Common Cause Principle:  $E_1 \leftarrow C \rightarrow E_2$



Hans Reichenbach 1891-1953

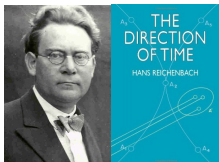
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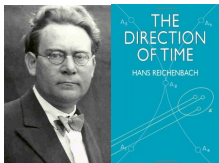
**Two-point** Correlation does not imply causation



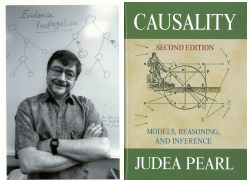
**Three-or-more-point** Correlation *might* imply causation

Common Cause Principle:  $E_1 \rightarrow M \rightarrow E_2$  ?

Independent 'Cause' Principle:  $C_1 \rightarrow E \leftarrow C_2$



Hans Reichenbach 1891-1953



# Can one Infer Causation from mere Correlation ?

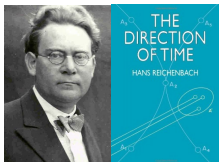
**Two-point** Correlation does not imply causation



**Three-or-more-point** Correlation *might* imply causation

Common Cause Principle:  $E_1 \rightarrow M \rightarrow E_2$  ?

But *latent variable* ?....  $C_1 \leftarrow L \rightarrow E \leftarrow C_2$



Hans Reichenbach 1891-1953

