Inference of Brain Dynamics

Using Reservoir Computing and Signatures for the Inference of Dynamic Causal Graphs

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Introduction

The use of machine learning models in neurology raises a key challenge : their **interpretability**. In a clinical context, it is essential to understand the mechanisms underlying predictions. **Dynamic causal graphs** provide a relevant approach by enabling the analysis of **directional relationships between brain regions in real time**, depending on the stimulus or task. They thus offer an interpretable framework to study the functional dynamics of the brain.

Hypothesis

The data under study consist of time series derived from EEG signals.

- The EEG signal describes a **trajectory** evolving on a **dynamic manifold**, whose geometry drifts over time and across individuals.
- Despite this variability, **invariant structures**, such as the topology of the manifold associated with the cognitive task, remain preserved for each individual.

We thus model the cognitive state space as a global manifold, where each task corresponds to an **attractor**. Cognitive transitions are then interpreted as continuous trajectories between basins of attraction.

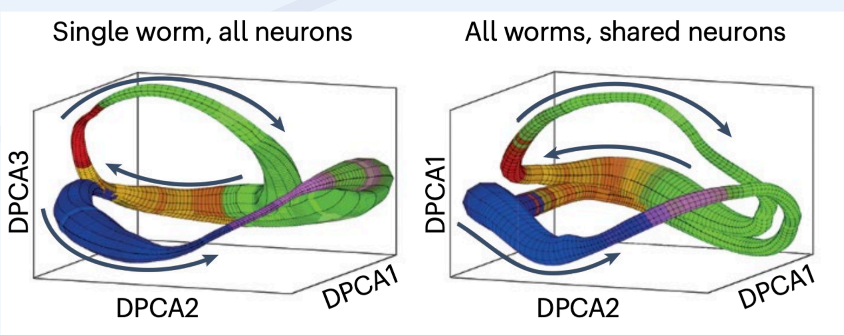


Figure – Neural manifold of a worm

Training

Step 1 - Feature Extraction

Extraction of geometric and temporal features from EEG signals using **path signatures**.

Let $X : [t-L, t] \to \mathbb{R}^d$ be a trajectory (a window of size L). Its truncated signature of order 2 is defined by iterated integrals :

$$\mathrm{Sig}^{(2)}_{t:t+L}(X) = \left(1, \underbrace{\int_t^{t+L} dX_u}_{\text{order 1}}, \underbrace{\int_t^{t+L} dX_{u_1} \otimes dX_{u_2}}_{\text{order 2}}\right)$$

Each EEG window is transformed into a path signature :

$$s_t = \mathrm{Sig}^{(m)}(X_{[t-L,t]})$$

where $s_t \in \mathbb{R}^{d_s}$ denotes the vector of geometric features extracted from the window $[t-L, t]$.

Step 2 - Reservoir Computing

- **Trained reservoir** : proxy for the dynamics of the input system.
- **Ultra-fast training** : only W_{out} is learned.

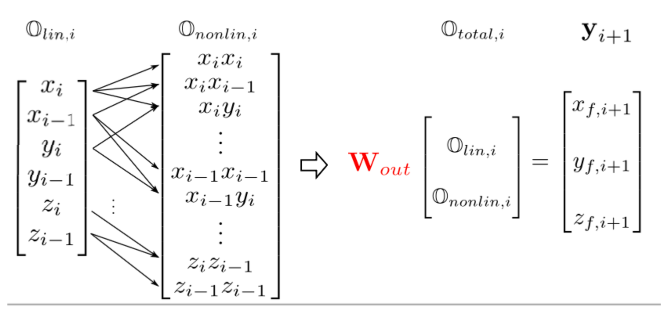


Figure – Reservoir Computing

The feature vector $z(t)$ is constructed from past EEG signal signatures, incorporating past windows and nonlinear interactions between signatures :

$$z(t) = \left[s(t), s(t-\tau), \dots, s(t-K\tau), \phi(s(t), \dots, s(t-K\tau))\right] \quad (1)$$

where ϕ denotes nonlinear functions.

The training consists in predicting the next value $s(t+1)$

$$y_t = \hat{s}_{t+1} = W_{\text{out}} \cdot z(t) + b$$

Causality - Dynamic Causal Graphs

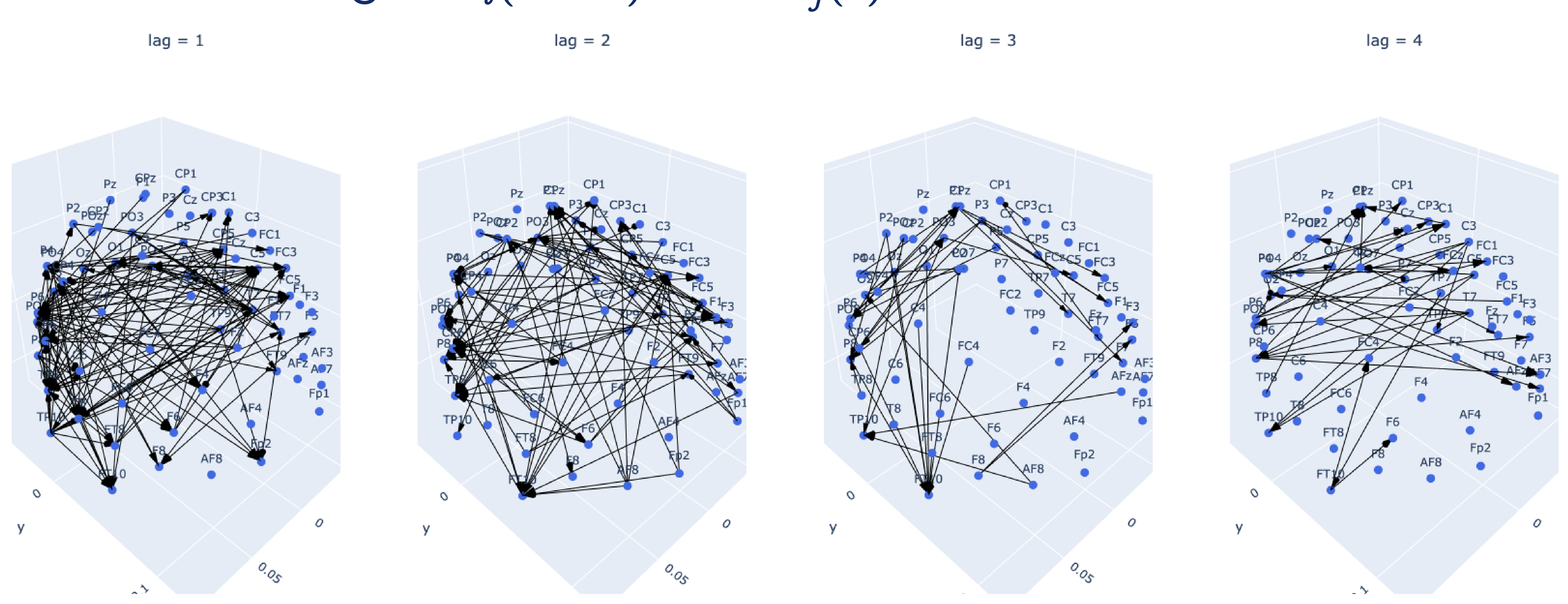
Step 3 - Inference of the Dynamic Causal Graph

We aim to estimate, at each time step, the causal graph between all pairs of electrodes.

In our formulation, the graph $G(t)$ is directly obtained from W_{out}, and the edge between i and j is defined as :

$$i \to j : G_{ij}(t) = \sum_k W_{ij,k}\, z_k(t) \quad (2)$$

Each coefficient $G_{ij}(t)$ mesures the contribution of explicit features z_t in the forcasting of $x_i(t+1)$ from $x_j(t)$.



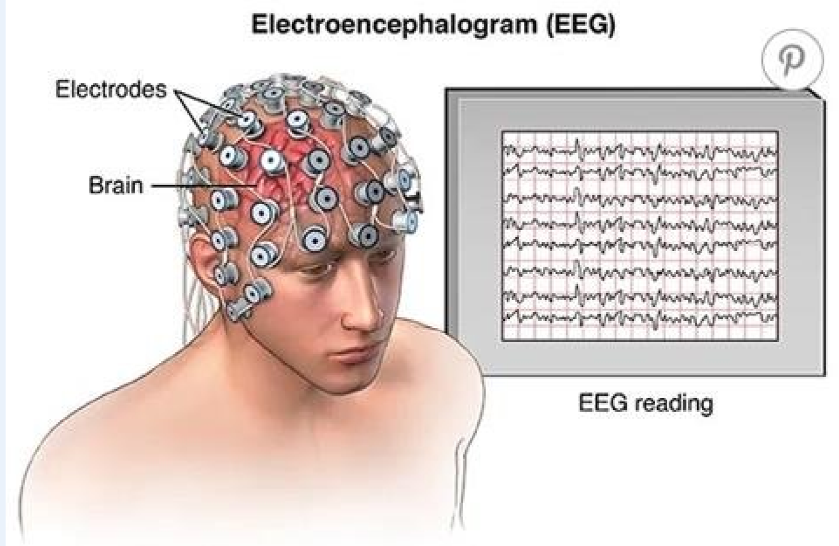


Figure – EEG

Discussion

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The main difficulty lies in the absence, to date, of an established reference causal graph for the brain, for a given task.

It is therefore necessary to evaluate our results by confronting them with domain expertise, or by analyzing to what extent the generated graphs are able to explain known phenomena.

Conclusion

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We propose an approach combining path signatures and reservoir computing to model the dynamics of EEG signals and infer dynamic causal graphs.

This framework captures both the geometry of trajectories and the directional interactions between brain regions. It opens new perspectives for a better understanding of cognitive states and their evolution over time.

References

- G. Preich, *A neural manifold view of the brain*, 2025
- Xiaoqi Xu, *Signature methods for brain-computer interfaces*, 2023
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